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Weak Nonlinear Thermal Instability Under Vertical Magnetic Field, Temperature Modulation And Heat Source

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Abstract
The present paper deals with a weak nonlinear stability problem of magneto-convection in an electrically conducting Newtonian liquid, confined between two horizontal surfaces, under a constant vertical magnetic field, and subjected to an imposed time-periodic boundary temperature (ITBT) along with internal heating effects. In the case of (ITBT), the temperature gradient between the walls of the fluid layer consists of a steady part and a time-dependent oscillatory part. The temperature of both walls is modulated in this case. The disturbance is expanded in terms of power series of amplitude of convection, which is assumed to be small. It is found that the response of the convective system to the internal Rayleigh number is destabilizing. Using Ginzburg-Landau equation, the effect of modulations on heat transport is analyzed. Effect of various parameters on the heat transport is also discussed. Further, it is found that the heat transport can be controlled by suitably adjusting the external parameters of the system.

I. INTRODUCTION

We know that, controlling convection is mainly concerned with space-dependent temperature gradients. There are many interesting situations of practical importance in which the temperature gradient is a function of both space and time. This uniform temperature gradient can be determined by solving the energy equation with suitable time-dependent thermal boundary conditions and can be used as an effective mechanism to control the convective flow. However, in practice, the non-uniform temperature gradient finds its origin in transient heating or cooling at the boundaries. Hence the basic temperature profile depends explicitly on position and time. This problem, called the thermal modulation problem, involves the solution of the energy equation under suitable time-dependent boundary conditions. Predictions exist for a variety of responses to modulation depending on the relative strength and rate of forcing. Among these, there is the upward or downward shift of convective threshold compared to the unmodulated problems. Lot of work is available in the literature covering how a time-periodic boundary temperature affects the onset of Rayleigh-Bénard convection. An excellent review related to this problem is given by Davis (1976).

The classical Rayleigh-Bénard convection due to bottom heating is well known and highly explored phenomenon given by Chandrasekhar (1961), Drazin and Reid (2004). Many researchers, under different physical models have investigated thermal instability in a horizontal fluid layer with temperature modulation. Some of them are: Venezian (1969), was the first to consider the effect of temperature modulation on thermal instability in a horizontal fluid layer. A similar problem was studied earlier by Gershuni and Zhukhovitskii (1963), for a temperature profile obeying rectangular law. Rosenblat and Herbert (1970), investigated the linear stability problem and found an asymptotic solution by considering low frequency modulation and free free surfaces. Rosenblat and Tanaka (1971), studied the linear stability for a fluid in a classical geometry of Bénard by considering the temperature modulation of rigid-rigid boundaries. The first nonlinear stability problem in a horizontal fluid layer, under temperature modulation of the boundaries was studied by Roppo et al. (1984). Bhadauria and Bhatia (2002), studied the effect of temperature modulation on thermal instability by considering rigid rigid boundaries and different types of temperature profiles. Bhadauria (2006), studied the effect of temperature modulation under vertical magnetic field by considering rigid boundaries. Malashetty and Swamy (2008), investigated thermal instability of a heated fluid layer subject to both boundary temperature modulation and rotation. Bhadauria et al. (2009), studied the non-linear aspects of thermal instability under temperature modulation, considering various temperature profiles. Raju and Bhattacharyya (2010), investigated onset of thermal instability in a horizontal layer of fluid with modulated boundary temperatures by considering rigid boundaries. Bhadauria et al.(2012) studied thermally or gravity modulated non-linear stability problem in a rotating viscous fluid layer, using Ginzburg-Landau equation for stationary mode of convection.

Thompson and Chandrasekhar (1951, 1961), were the first to study the magneto-convection in horizontal fluid layer. Nakagawa (1955, 1957) and Jirlow (1956), found that, vertical magnetic field suppresses the onset of convection by using Galerkin method. Finlayson (1970), studied the problem of magneto-convection in a horizontal layer of magnetic fluid which is heated from below and cooled from above. Bhatia and Steiner (1973), found that a magnetic field has a stabilizing effect on thermal instability. Gotoh and Yamada (1982), studied the problem of magneto-convection in a horizontal layer of magnetic fluid which is heated from below and cooled from above and found condition for onset of convection. Oreper and Szekely (1983), have found that the presence of a magnetic field can suppress natural convection currents and that the strength of the magnetic field is one of the important factors in determining the quality of the crystal. Bajaj and Malik (1997, 1998) examined the stability of various flow patterns while studying Rayleigh-Bénarsd convection in magnetic fluids. The effect of a magnetic field on free or natural convection in a rectangular enclosure having isothermal and adiabatic walls were studied by Garandet et al. (1992), Rudraiah et al.(1995) and Al-Najem et al. (1998). Siddheshwar and Pranesh (1999, 2000), examined the effect of a transverse magnetic field on thermal/gravity convection in a weak electrically conducting fluid with internal angular momentum. Siddheshwar and Pranesh (2002), analyzed the role of magnetic field in the inhibition of natural convection driven by combined buoyancy and surface tension forces in a horizontal layer of an electrically conducting Boussinesq fluid with suspended particles confined between an upper free/adiabatic and a lower rigid/isothermal boundary is considered in 1g and μg situations. Kaddeche et al. (2003) have investigated the buoyant convection induced between infinite horizontal walls by horizontal temperature gradient. Bhadauria et al. (2008, 2010), also studied the effect of magnetic field on thermal modulated convection in the case of porous medium. Bhadauria and Sherani (2008, 2010), investigated onset of Darcy-convection in a magnetic fluid-saturated porous medium subject to temperature modulation of the boundaries and magneto-convection in a porous medium under temperature modulation. Siddheshwar et al. (2012) performed a local non-linear stability analysis of Rayleigh-Bénard magneto-convection using Ginzburg-Landau equation. They showed that gravity modulation can be used to enhance or diminish the heat transport in stationary magneto-convection.

The above studies on thermal instability under modulation are made for non-internal heating system. However, in many situations of great practical importance, it is found that, the material offers its own source of heat, and this leads to a different way in which a convective flow can be set up through the local heat generation within the layer. Such a situation can occur through radioactive decay or relatively weak exothermic reaction and nuclear reaction which can take place within the material. It is the main source of energy for celestial bodies caused by nuclear fusion and decaying of radioactive materials, which keeps the celestial objects warm and active. Due to internal heat there exist a thermal gradient between the interior and exterior of the earth's crust, saturated by multi component fluids, which helps convective flow, thereby transforming the thermal energy towards the surface of the earth. Therefore the role of internal heat generation becomes very important in several applications including storage of radioactive materials, combustion and fire studies, geophysics, reactor safety analysis and metal waste form development for spent nuclear fuel. However there are very few studies available in which the effect of internal heating on convective flow in a fluid layer has been investigated. They are due to; Roberts (1967), Tveitereid and Palm (1976), Tveitereid (1978), Yu and Shih (1980), Bhattacharya and Jena (1984), Takashima (1989), Tasaka and Takeda (2005), Joshi et al.(2006), Bhadauria (2012) and Bhadauria et al. (2013).

Here the first paragraph talks about introduction of temperature modulation, second paragraph on temperature modulation in a fluid layer, third paragraph on magneto-convection and above paragraph on the effect of internal heat source on thermal convection. Every paragraph has its own meaning but, not much modulation work has done in the case of magneto-convection. Recently Siddheshwar et al. (2012), studied a weakly non-linear magneto convection under temperature and gravity modulation without internal heat source. Due to this, in this paper we have investigated internal heating effects on magneto-convection under temperature modulation.

II. GOVERNING EQUATIONS

We consider an electrically conducting liquid of depth d, confined between two infinite, parallel, horizontal planes at z=0 and z=d. Cartesian co-ordinates have been taken with the origin at the bottom of the liquid layer, and the z-axis vertically upwards. The surfaces are maintained at a constant gradient $\nabla T / d$ and a constant magnetic field $H_b \hat{k}$ is applied across the liquid region (given in Fig.1).

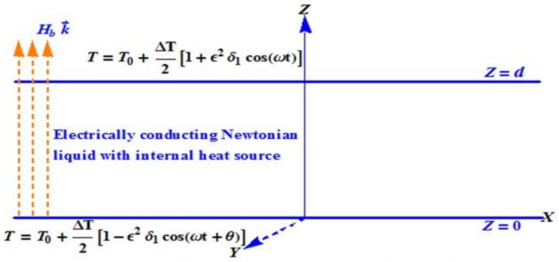

Fig .1 : Physical configuration of the problem

Under the Boussinesq approximation, the dimensional governing equations for the study of magneto-convection in an electrically conducting liquid are:

$$\nabla\cdot\vec{q}=0 \tag{1}$$

$$\nabla\cdot\vec{H}=0, \tag{2}$$

$$\frac{\partial\vec{q}}{\partial t}+(\vec{q}.\nabla)\vec{q}=-\frac{1}{\rho_0}\nabla p+\frac{\rho}{\rho_0}\vec{g}-\frac{\mu}{\rho_0}\nabla^2\vec{q}+\frac{\mu_m}{\rho_0}\vec{H}.\nabla\vec{H} \tag{3}$$

$$\gamma\frac{\partial T}{\partial t}+(\vec{q}\cdot\nabla)T=\kappa_T\nabla^2T+Q(T-T_0), \tag{4}$$

$$\frac{\partial\vec{H}}{\partial t}+(\vec{q}.\nabla)\vec{H}-(\vec{H}.\nabla)\vec{q}=\nu_m\nabla^2\vec{H} \tag{5}$$

$$\rho=\rho_0[1-\beta_T(T-T_0)], \tag{6}$$

where $\vec{q}$ is velocity (u, v, w), Q is internal heat source, μ fluid viscosity, κ_T is the thermal diffusivity, T is temperature, β_T is thermal expansion coefficient, γ is the ratio of heat capacities. For simplicity of the problem it is considered to be one in this paper, ρ is the density,

$\vec{g}=(0,0,-\mathrm{g})$ is the acceleration due to gravity, while ρ_0 is the reference density and $\vec{H}$ magnetic field. The exeternally imposed wall temperature conditions given by Venezian (1969):

$$T=T_0+\frac{\Delta T}{2}\left[1+\varepsilon^2\delta_1\cos(\omega t)\right] \quad \text{at z=0}$$

$$=T_0-\frac{\Delta T}{2}\left[1-\varepsilon^2\delta_1\cos(\omega t+\theta)\right] \text{ at z=d} \tag{7}$$

where δ_1 small amplitude of temperature modulation, ΔT is the temperature difference across the fluid layer, ω is modulation frequency and θ is the phase difference. The basic state is assumed to be quiescent and the quantities in the state are given by:

$$\vec{q}_b(z)=0,\ \rho=\rho_b(z,t),\ p=p_b(z,t),\ T=T_b(z,t) \tag{8}$$

$$\frac{\partial p_b}{\partial z}=-\vec{g}\rho_b, \tag{9}$$

$$\frac{\partial T_b}{\partial t}=\kappa_T\frac{\partial^2T_b}{\partial z^2}+Q(T_b-T_0), \tag{10}$$

$$\rho_b=\rho_0[1-\beta_T(T_b-T_0)]. \tag{11}$$

For basic state temperature field given in Eq.(10), has been solved subject to the thermal boundary conditions (7), and the solution is found to be of the form

$$T_b(z,t)=T_s(z)+\varepsilon^2\delta_1\,\mathrm{Re}\{T_1(z,t)\}, \tag{12}$$

where $T_s(z)$ is the steady temperature field and T_1 is the oscillating part, while Re stands for the real part.

We impose finite amplitude perturbations on the basic state in the form:

$$\vec{q}=q_b+\vec{q}',\ \rho=\rho_b+\rho',\ p=p_b+p',\ T=T_b+T' \tag{13}$$

Substituting Eq.(13) into Eqs.(1) - (6) and using the basic state results, we get the following equations

$$\nabla\cdot\vec{q}'=0\ , \tag{14}$$

$$\nabla\cdot\vec{H}'=0\ , \tag{15}$$

$$\frac{\partial\vec{q}'}{\partial t}+(\vec{q}'.\nabla)\vec{q}'-\frac{\mu_m}{\rho_0}(\vec{H}'.\nabla)\vec{H}'=-\frac{1}{\rho_0}\nabla p'-\frac{\rho'}{\rho_0}g\hat{k}+\nu\nabla^2\vec{q}'+\frac{\mu_m}{\rho_0}H_b\frac{\partial\vec{H}'}{\partial z} \tag{16}$$

$$\frac{\partial T'}{\partial t}+(\vec{q}'\cdot\nabla)T''+w'\frac{\partial T'}{\partial z}=\kappa_T\nabla^2T'+QT'\ , \tag{17}$$

$$\frac{\partial\vec{H}'}{\partial t}+(\vec{q}'.\nabla)\vec{H}'-(\vec{H}'.\nabla)\vec{q}'-H_b\frac{\partial w'}{\partial z}=\nu_m\nabla^2\vec{H}' \tag{18}$$

$$\rho'=\rho_0\beta_TT'. \tag{19}$$

Further we consider only two dimensional disturbances in our study and hence the stream function ψ and magnetic potential Φ are introduced as $(u',w')=\left(\frac{\partial\psi}{\partial z},-\frac{\partial\psi}{\partial x}\right)$ and $(\vec{H}_x',\vec{H}_z')=\left(\frac{\partial\Phi}{\partial z},-\frac{\partial\Phi}{\partial x}\right)$. We eliminate density and presser terms from the Eqs.(16-19), and the resulting system nondimensionalized using the following transformations:

$$(x',y',z')=d(x^*,y^*,z^*),\ t'=\frac{d^2}{\kappa_T}t^*$$

$$q'=\frac{\kappa_T}{d}q^{*}{}'\ \psi=\kappa_T\psi^*\ T'=\Delta T\ T^* \text{ and}$$

$$\omega^*=\frac{\kappa_T}{d^2}\omega,\qquad \Phi=d\vec{H}_b\Phi^*,$$ we get the non-dimensional governing equations in the form:

$$-\nabla^4\psi + Ra_T\frac{\partial T}{\partial x} - QP_m\frac{\partial}{\partial z}\nabla^2\Phi = -\frac{1}{\Pr}\frac{\partial}{\partial t}\nabla^2\psi + \frac{1}{\Pr}\frac{\partial(\psi,\nabla^2\psi)}{\partial(x,z)} - QP_m\frac{\partial(\Phi,\nabla^2\Phi)}{\partial(x,z)}, \tag{20}$$

$$-\frac{\partial\psi}{\partial x}\frac{\partial T_b}{\partial z} - (\nabla^2 + \mathrm{R}_i)T = -\frac{\partial T}{\partial t} + \frac{\partial(\psi,\mathrm{T})}{\partial(\mathrm{x},\mathrm{z})}, \tag{21}$$

$$-\frac{\partial\psi}{\partial z} - \mathrm{Pm}\,\nabla^2\Phi = -\frac{\partial\Phi}{\partial t} + \frac{\partial(\psi,\Phi)}{\partial(x,z)}. \tag{22}$$

Here the non-dimensionalizing parameters in the above equations are given in Nomenclature. Equation (21) shows that, the basic state solution influences the stability problem through the factor $\partial T_b / \partial z$ which is given by:

$$\frac{\partial T_b}{\partial z} = f_1(\mathrm{z}) + \varepsilon^2\delta_1[f_2(z,t)], \tag{23}$$

Where

$$f_1(z) = -\frac{\sqrt{R_i}}{2sin\sqrt{R_i}}\left(cos\sqrt{R_i}(1-\mathrm{z}) + \cos\sqrt{R_i}(z)\right) \tag{24}$$

$$f_2 = \mathrm{Re}[fe^{-i\omega t}], \tag{25}$$

$$f(z) = [A(m)e^{mz} + A(-m)e^{-mz}],$$

$$A(\lambda) = \frac{m}{2}\frac{e^{-i\theta} - e^{-m}}{e^m - e^{-m}},\ m = \sqrt{\lambda^2 - R_i},\ \lambda^2 = -i\omega.$$

To keep the time variation slow, we have rescaled the time t by using the time scale $\tau = \varepsilon^2$ t. Here, we study only stationary solution, therefore overstable solutions are not considered. Now, to study the stationary convection, we write the non-linear Eqs.(20)-(22) in the matrix form as given below:

$$\begin{bmatrix} -\nabla^4 & Ra_T\frac{\partial}{\partial x} & -QPm\frac{\partial}{\partial z}\nabla^2 \\ f_1\frac{\partial}{\partial x} & -(\nabla^2 + R_i) & 0 \\ -\frac{\partial}{\partial z} & 0 & -Pm\nabla^2 \end{bmatrix}\begin{bmatrix}\psi \\ T \\ \Phi\end{bmatrix} = \begin{bmatrix} -\frac{\varepsilon^2}{\Pr}\frac{\partial\nabla^2\psi}{\partial\tau} + \frac{1}{\Pr}\frac{\partial(\psi,\nabla^2\psi)}{\partial(x,z)} - QPm\frac{\partial(\Phi,\nabla^2\Phi)}{\partial(x,z)} \\ -\varepsilon^2\frac{\partial T}{\partial\tau} + \varepsilon^2\delta_1 f_2\frac{\partial\psi}{\partial x} + \frac{\partial(\psi,T)}{\partial(x,z)} \\ -\varepsilon^2\frac{\partial\Phi}{\partial\tau} + \frac{\partial(\psi,\Phi)}{\partial(x,z)} \end{bmatrix} \tag{26}$$

The considered boundary conditions to solve equation (26) are:

$$\psi = \nabla^2\psi = 0,\ \text{and}\ \Phi = D\Phi = 0 \quad \text{on} \quad \text{z=0}$$
$$\psi = \nabla^2\psi = 0,\ \text{and}\ \Phi = D\Phi = 0 \quad \text{on} \quad \text{z=1} \tag{27}$$

wher $D = \partial / \partial z$.

III. FINITE AMPLITUDE EQUATION AND HEAT TRANSPORT FOR STATIONARY INSTABILITY

We introduce the following asymptotic expansion in Eq.(26):

$$\mathrm{R}a_T = R_{0c} + \varepsilon^2 R_2 + \ldots$$

$$\psi = \varepsilon\psi_1 + \varepsilon^2\psi_2 + \varepsilon^3\psi_2 + \ldots.$$
$$T = \varepsilon T_1 + \varepsilon^2 T_2 + \varepsilon^3 T_3 + ..$$
$$\Phi = \varepsilon\Phi_1 + \varepsilon^2\Phi_2 + \varepsilon^3\Phi_3 + .. \tag{28}$$

where R_{0c} is the critical Rayleigh number at which the onset of convection takes place in the absence of modulation. Now substituting Eqn (28) we solve the system Eq.(26) for different orders of ε. **At the first order,** we have

$$\begin{bmatrix} -\nabla^4 & R_{0c}\frac{\partial}{\partial x} & -QPm\frac{\partial}{\partial z}\nabla^2 \\ f_1\frac{\partial}{\partial x} & -(\nabla^2 + R_i) & 0 \\ -\frac{\partial}{\partial z} & 0 & -Pm\nabla^2 \end{bmatrix}\begin{bmatrix}\psi_1 \\ T_1 \\ \Phi_1\end{bmatrix} = \begin{bmatrix}0 \\ 0 \\ 0\end{bmatrix} \tag{29}$$

The solution of the lowest order system subject to the boundary conditions Eq.(27), is

$$\psi_1 = A(\tau)\sin(k_c\,\mathrm{x})\sin(\pi\,\mathrm{z}) \tag{30}$$

$$T_1 = -\frac{4\pi^2 k_c}{\delta_R^2(4\pi^2 - R_i)}A(\tau)\cos(k_c x)\sin(\pi z), \tag{31}$$

$$\Phi_1 = \frac{\pi}{Pm\delta^2}A(\tau)\sin(k_c\,\mathrm{x})\cos(\pi\,\mathrm{z}), \tag{32}$$

where $\delta^2 = \mathrm{k}_c^2 + \pi^2$, $\delta_R^2 = \delta^2 - R_i$. The critical value of the Rayleigh number for the onset of stationary convection is calculated numerically, and the expression is given by:

$$R_{0c} = \frac{\delta_R^2(\delta^2 + Q\pi^2)(4\pi^2 - R_i)}{4\pi^2 k_c^2}, \tag{33}$$

At the second order, we have:

$$\begin{bmatrix} -\nabla^4 & R_{0c}\frac{\partial}{\partial x} & -QPm\frac{\partial}{\partial z}\nabla^2 \\ f_1\frac{\partial}{\partial x} & -(\nabla^2 + R_i) & 0 \\ -\frac{\partial}{\partial z} & 0 & -Pm\nabla^2 \end{bmatrix}\begin{bmatrix}\psi_2 \\ T_2 \\ \Phi_2\end{bmatrix} = \begin{bmatrix}R_{21} \\ R_{22} \\ R_{23}\end{bmatrix} \tag{34}$$

$$R_{21} = 0, \tag{35}$$

$$R_{22} = \frac{\partial\psi_1}{\partial x}\frac{\partial T_1}{\partial z} - \frac{\partial T_1}{\partial x}\frac{\partial\psi_1}{\partial z}, \tag{36}$$

$$R_{23} = \frac{\partial\psi_1}{\partial x}\frac{\partial T_1}{\partial z} - \frac{\partial T_1}{\partial x}\frac{\partial\psi_1}{\partial z}. \tag{37}$$

The second order solutions subjected to the boundary conditions Eq. (27) is obtained as follows:

$$\psi_2 = 0, \tag{38}$$

$$T_2 = -\frac{2\pi^3 k_c^2}{\delta_R^2(4\pi^2 - R_i)^2}A^2(\tau)\sin(2\pi z), \tag{39}$$

$$\Phi_2 = \frac{\pi^2}{8k_c Pm^2\delta^2} A^2(\tau)\sin(2k_c\, x). \tag{40}$$

The horizontally-averaged Nusselt number Nu, for the stationary mode of convection is given by:

$$Nu(\tau) = 1 + \frac{\left[\frac{k_c}{2\pi}\int_0^{\frac{2\pi}{k_c}} \frac{\partial T_2}{\partial z}dx\right]_{z=0}}{\left[\frac{k_c}{2\pi}\int_0^{\frac{2\pi}{k_c}} \frac{\partial T_b}{\partial z}dx\right]_{z=0}} \tag{41}$$

$$Nu(\tau) = 1 + \frac{8\pi^4 k_c^2 \sin\sqrt{R_i}}{\delta_R^2(4\pi^2 - R_i)^2\sqrt{R_i}(\cos\sqrt{R_i}+1)} A^2(\tau). \tag{42}$$

We must note here that f_2 is effective at $O(\varepsilon^2)$ and affects Nu(τ) through $A(\tau)$ as in Eq.(42).

At the third order, we have

$$\begin{bmatrix} -\nabla^4 & R_{0c}\frac{\partial}{\partial x} & -QPm\frac{\partial}{\partial z}\nabla^2 \\ f_1\frac{\partial}{\partial x} & -(\nabla^2 + R_i) & 0 \\ -\frac{\partial}{\partial z} & 0 & -Pm\nabla^2 \end{bmatrix} \begin{bmatrix} \psi_3 \\ T_3 \\ \Phi_3 \end{bmatrix} = \begin{bmatrix} R_{31} \\ R_{32} \\ R_{33} \end{bmatrix} \tag{43}$$

where

$$R_{31} = -\frac{1}{\Pr}\frac{\partial}{\partial\tau}\nabla^2\psi_1 - R_2\frac{\partial T_1}{\partial x} - QPm\left(\frac{\partial\Phi_1}{\partial z}\frac{\partial}{\partial x}\nabla^2\Phi_2 - \frac{\partial\Phi_2}{\partial x}\frac{\partial}{\partial z}\nabla^2\Phi_1\right), \tag{44}$$

$$R_{32} = -\frac{\partial T_1}{\partial\tau} + \frac{\partial T_2}{\partial z}\frac{\partial\psi_1}{\partial x} + \delta_1 f_2\frac{\partial\psi_1}{\partial x}, \tag{45}$$

$$R_{33} = -\frac{\partial\Phi_1}{\partial\tau} + \frac{\partial\Phi_2}{\partial x}\frac{\partial\psi_1}{\partial z}. \tag{46}$$

Substituting ψ_1, T_1, Φ_1 and Φ_2 into Eqs.(44)-(46), and applying the solvability condition for the existence of the third order solution, we get the Ginzburg-Landau equation in the form

$$A_1\frac{\partial A(\tau)}{\partial\tau} - A_2 A(\tau) + A_3 A(\tau)^3 = 0\,. \tag{47}$$

where

$$A_1 = \frac{\delta^2}{\Pr} + \frac{\delta^4 + Q\pi^2}{\delta_R^2} - \frac{Q\pi^2}{Pm\delta^2},$$

$$A_2 = (\delta^4 + Q\pi^2)\left(\frac{R_2}{R_{0c}} - \frac{4\pi^2 - R_i}{2\pi^2}\delta_1 I_1\right),$$

$$A_3 = \frac{(\delta^4 + Q\pi^2)k_c^2\pi^2}{2\delta_R^2(4\pi^2 - \mathrm{R}_i)} + \frac{Qk_c^2\pi^4}{2Pm^2\delta^4} - \frac{Q\pi^4}{4Pm^2\delta^2}, \text{ and}$$

$$I_1(\tau) = \int_{z=0}^{1} f_2\sin^2(\pi z)dz.$$

It may be difficult to get the analytic solution of the above Ginzburg-Landau equation (47) due to its non-autonomous nature, therefore, it has been solved numerically using the inbuilt function NDSolve of Mathmatica 8.0, subject to the suitable initial condition B(0) = a0, where a0 is the chosen initial amplitude of convection. In our calculations we may assume $R_2 = R_{0c}$ to keep the parameters to the minimum

IV. RESULTS AND DISCUSSION

In this paper, the combined effect of internal heating and temperature modulation on thermal instability in a electrically conducting fluid layer. A weakly non-linear stability analysis has been performed to investigate the effect of temperature modulation on heat transport. The effect of temperature modulation on the Rayleigh-Bénard system has been assumed to be of order $O(\varepsilon^2)$. This means, we consider only small amplitude temperature modulation. Such an assumption will help us in obtaining the amplitude equation of convection in a rather simple and elegant manner and is much easier to obtain than in the case of the Lorenz model.

In Phase Modulation

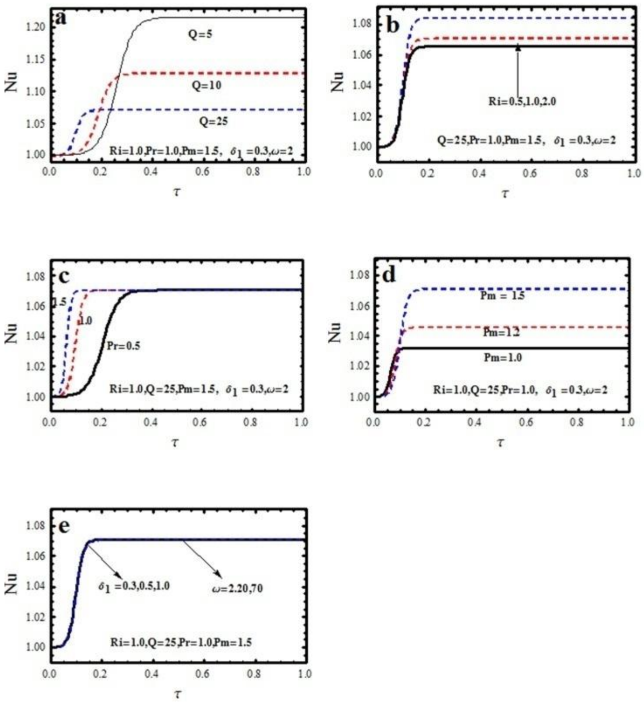

Fig 2. Nu versus τ for different values of system parameters

We give the following features of the problem before our results: In the basic state, heat transport is by conduction alone. We consider the following three types of temperature modulation on magneto-convection:

1. In-phase modulation (IPM) (θ =0),
2. Out-of-phase modulation (OPM) (θ=л) and
3. Modulation of only the lower boundary (LBMO) (θ=-i∞).

In this case the modulation effect will not be considered in the upper boundary, but only in the lower boundary. The parameters that arise in the problem are Q, Pr, Pm, R_i, θ, δ_1, ω these parameters influence the convective heat and mass transfer. The first five parameters related to the fluid layer, and the last three concern the external mechanisms of controlling convection.

Out of phase modulation

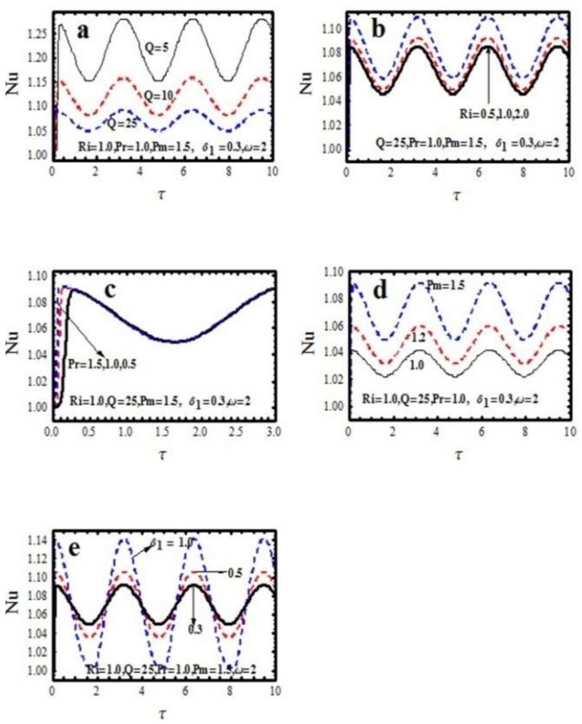

Fig 3. Nu versus τ for different values of system parameters

The effect of temperature modulation is represented by amplitude δ_1 which lies around 0.3. The effect of electrical conductivity and magnetic field comes through Pm, Q. There is the property of the fluid coming into picture as well as through Prandtl number Pr. Further, the modulation of the boundary temperature assumed to be of low frequency. At low range of frequencies the effect of frequency on onset of convection as well as on heat transport is minimal. This assumption is required in order to ensure that the system does not pick up oscillatory convective mode at onset due to modulation in a situation that is conductive otherwise to stationary mode. It is important at this stage to consider the effect of Q, Pr, Pm, R_i, θ, δ_1, ω on the onset of convection. The heat transfer quantified by the Nusselt numbers which is given in Eq.(42). The figures (2-4) show that, the individual effect of each non-dimensional parameter on heat transfer.

1. The Chandrasekhar number Q which is ratio of Lorentz force to viscous force where, the force exerted on a charged particle moving with velocity through an electric and magnetic field. The entire electromagnetic force on the charged particle is called the Lorentz force. As Q increases Lorentz force dominates viscous forces and the result is to delay the onset of convection, hence heat transfer. The Nusselt number Nu starts with one by showing conduction state, and for small values of time τ increases and becomes constant for large values of time τ in the case of (IPM) given in (2a). In the case of (OPM 3a, LBMO 4a) the effect of Q shows oscillatory behavior and increment in Q decreases the magnitude of Nu. Hence Q has stabilizing effect in all the three types of modulations so that heat transfer decrease with Q.
2. The effect of internal Rayleigh number R_i is to increase Nu so that heat transfer. Hence it has destabilizing effect in all the three types of modulations which is given by the figures (2b, 3b, 4b).
3. In OPM case an increment in Prandtl number Pr for slow time scale there is sudden increment in Nu which shows advances of convection and hence heat transfer, but, for large values of time study sate. An oscillatory behavior obtained in the case of (OPM 3c, LBMO 4c).
4. The effect of Magnetic Prandtl Pm numbers is to advances the convection and heat transfer. Hence Pm has destabilizing effect of the system given in figures (2d) IPM case. In the case of (OPM 3d, LBMO 4d) the effect of Pm shows oscillatory behavior and increment in Pm increases the magnitude of Nu. Hence Pm has destabilizing effect hence heat transfer.
5. In the case of (IPM) we observe that no effect of amplitude δ_1, and frequency ω of modulation which is given by the figure (2e). But in the case of (OPM 3e, LBMO 4e) the increment in δ_1, leads to increment in magnitude of Nu hence heat transfer. The increment in ω shortens the wavelength but no effect in magnitude so we are not presenting here as figure.
6. The comparison of three types of temperature modulations given in figure (4f).

$$[Nu]^{IPM} < [Nu]^{LBMO} < [Nu]^{OPM} .$$

V. CONCLUSIONS

The effects of temperature modulation and internal heating on Rayleigh-Bénard convection in an electrically conducting fluid layer have been analyzed by performing a weakly nonlinear stability analysis resulting in the real Ginzburg-Landau amplitude equation. The following conclusions are drawn:

Only lower boundary modulation

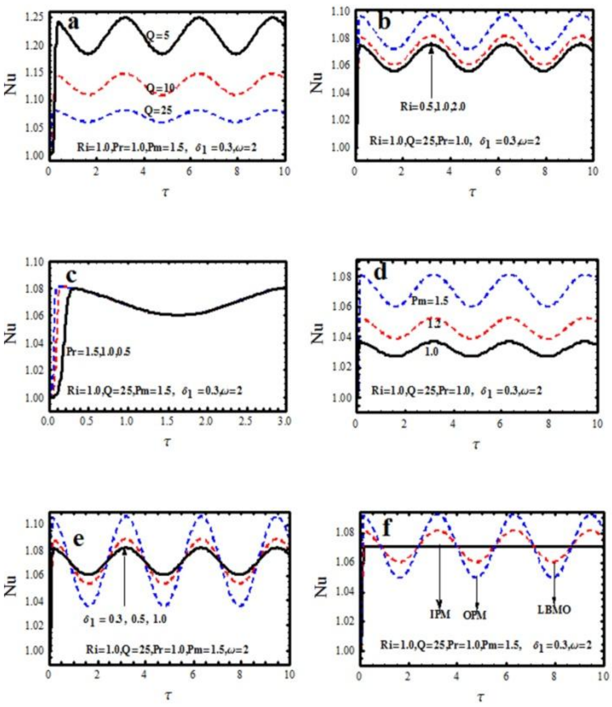

Fig 4. Nu versus τ for different values of system parameters

1. The effect of IPM is negligible on heat transport in the system.
2. In the case of IPM, the effect of δ_1 and ω are also found to be negligible on heat transport.
3. In the case of IPM, the values of Nu increase steadily for small values of time τ, however become constant when τ is large.
4. The effect of increasing Pr, Pm, R_i is found to increase in Nu thus increasing heat transfer for all three types of modulations.
5. The effect of increasing δ_1 is to increase the value of Nu for the case of OPM and LBMO, hence heat transfer.
6. The effect of increasing ω is to decrease the value of Nu for the case of OPM and LBMO, hence heat transfer.
7. In the cases of OPM and LBMO, the nature of Nu remains oscillatory.
8. Initially when τ is small, the values of Nusselt number start with 1, corresponding to the conduction state. However as τ increases, Nu also increase, thus increasing the heat transfer.
9. The values of Nu for LBMO are greater than those in IPM but smaller than those in OPM.
10. The effect of magnetic field is to stabilize the system.

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Nomenclature

Latin Symbols

A	Amplitude of convection
δ_1	Amplitude of Temperature modulation
d	Depth of the fluid layer
$\vec{g}$	Acceleration due to gravity
k_c	Critical wave number
Nu	Nusselt number
p	Reduced pressure
R_i	Internal heat source parameter $R_i = Qd^2/\kappa_T$
Ra_T	Thermal Rayleigh number, $Ra_T = \dfrac{\beta_T g \Delta T d^3}{\nu \kappa_T}$
R_{0c}	Critical Rayleigh number
T	Temperature
Pr	Prandtl number $\Pr = \dfrac{\nu}{\kappa_T}$
Pm	Magnetic Prandtl number $Pm = \dfrac{\nu_m}{\kappa_T}$
ΔT	Temperature difference across the fluid layer
t	Time
(x; z)	Horizontal and vertical co-ordinates

Greek Symbols

β_T	Coefficient of thermal expansion
ν_m	Magnetic viscosity
δ^2	Square of horizontal wave number
ε	Perturbation parameter
κ_T	Effective thermal diffusivity
ω	Frequency of modulation
μ	Dynamic viscosity of the fluid
μ_m	Magnetic permeability
ν	Kinematic viscosity,
ρ	Fluid density
ψ	Stream function
Φ	Magnetic potential
τ	Slow time
T'	Perturbed temperature
θ	Phase angle

Other symbol

∇^2	$\dfrac{\partial^2}{\partial x} + \dfrac{\partial^2}{\partial y} + \dfrac{\partial^2}{\partial z}$

Subscripts

b	Basic state
c	Critical
0	Reference value

Subscripts

‘	Perturbed quantity
*	Dimensionless quantity